MA114 Summer 2018
Worksheet 5 – Approximate Integration
6/14/18

1. (a) Write down the Midpoint rule and illustrate how it works with a sketch.

   (b) How large should n be in the Midpoint rule so that you can approximate

   $$\int_0^1 \sin(x)\,dx$$

   with an error less than 10^{-7}?

2. Use the midpoint rule to approximate the value of $\int_{-1}^{1} e^{-x^2}\,dx$ with $n = 4$. Draw a sketch to determine if the approximation is an overestimate or underestimate of the integral.

3. Draw the graph of $f(x) = \sin(\frac{1}{2}x^2)$ in the region $[0, 1]$ by $[0, 0.5]$ and let $I = \int_0^1 f(x)\,dx$.

   (a) Use the graph to decide whether L_2, R_2, M_2, and T_2 underestimate or overestimate I.

   (b) For any value of n, list the numbers L_n, R_n, M_n, T_n, and I in increasing order.

   (c) Compute L_5, R_5, M_5, and T_5. From the graph, which do you think gives the best estimate of I?

4. The velocity in meters per second for a particle traveling along the axis is given in the table below. Use the Midpoint rule and Trapezoid rule to approximate the total displacement of the particle from $t = 0$ to $t = 6$.

t	$v(t)$
0	0.75
1	1.34
2	1.5
3	1.9
4	2.5
5	3.2
6	3.0

5. Approximate the integral $\int_1^2 \frac{1}{x}\,dx$ using Simpson's rule. Choose n so that your error is certain to be less than 10^{-3}. Compute the exact value of the integral and compare it to your approximation.